In Table 1 the Madelung constants for NaCl, CsCl, ZnS, CaF_2 are listed to better than 12 decimals with different normalizations.

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1. K. HAYASHI, *Tafeln der Besselschen, Theta-, Kugel-, und anderer Funktionen*, Springer, Berlin, 1930.

2. O. EMERSLEBEN, "Numerische Werte des Fehlerintegrals für $\sqrt{n\pi}$," *Z. Angew. Math. Mech.*, v. 31, 1951, pp. 393–394.

EDITORIAL NOTE: For reviews of earlier, related papers by the same author, see *MTAC*, v. 5, 1951, pp. 77–78, RMT **871**; v. 11, 1957, pp. 109–110, RMT **56**; *ibid.*, p. 113–114, RMT **65**.

103[L, X].—LUCY JOAN SLATER, *Generalized Hypergeometric Functions*, Cambridge University Press, New York, 1966, xiii + 273 pp., 24 cm. Price $13.50.

The author of this valuable book remarks in the preface that the volume should really be attributed to both the late W. N. Bailey and Miss Slater. It was Professor Bailey's intention to write a comprehensive book on hypergeometric functions with the assistance of Miss Slater. The present work is dedicated to the memory of W. N. Bailey and is based in part on notes for a series of lectures which he gave during the years 1947–1950.

The ordinary hypergeometric or Gauss series is

$$\sum_{k=0}^{\infty} \frac{(a)_k(b)_k z^k}{(c)_k k!}, \qquad |z| < 1, \qquad (a)_k = \Gamma(a+k)/\Gamma(a),$$

and is usually represented by the symbol ${}_2F_1(a, b, c; z)$. A natural generalization is a series like the above but with an arbitrary number of numerator and denominator parameters. Thus (formally at least)

$${}_pF_q\left(\begin{matrix} a_1, a_2, \cdots, a_p \\ b_1, b_2, \cdots, b_q \end{matrix}\middle| z\right) = \sum_{k=0}^{\infty} \frac{(a_1)_k(a_2)_k \cdots (a_p)_k z^k}{(b_1)_k(b_2)_k \cdots (b_q)_k k!}.$$

The latter is called a generalized Gauss function or hypergeometric function, and where no confusion can result, is simply called a ${}_pF_q$. If in the above ${}_pF_q$, a numerator parameter is set to unity and the summation index sums from $-\infty$ to ∞, then such a series is called a bilateral series. It obviously can be expressed as a combination of two generalized hypergeometric functions.

The ${}_pF_q$ may be generalized by considering an obvious generalization of the double series

$$\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(a)_{m+n}(b)_m(b')_m}{(c)_{m+n}\, m!n!} x^m y^m$$

to an arbitrary number of numerator and denominator parameters. These are known as Appell series or functions. We can also have triple, quadruple, or multiple sums which are known as Lauricella functions.

The theory of the ${}_pF_q$ is fundamental in the applications since many of the special functions (Bessel functions, Legendre functions, etc.) are special cases. Appell and Lauricella functions also appear in the applications.

A different view of the problem of generalizing the Gauss function was adopted by E. Heine. He defined a basic number as $a_q \equiv (1 - q^a)/(1 - q)$. Observe that $a_q \to a$ as $q \to 1$. Heine defined the basic analogue of the Gauss series as

$$ {}_2\Phi_1(a, b; c; q, z) = 1 + \frac{(1 - q^a)(1 - q^b)z}{(1 - q^c)(1 - q)} + \frac{(1 - q^a)(1 - q^{a+1})(1 - q^b)(1 - q^{b+1})z^2}{(1 - q^c)(1 - q^{c+1})(1 - q)(1 - q^2)} + \cdots, $$

$|q| < 1$, so when $q \to 1$, the latter approaches ${}_2F_1$. The Heine series and its natural extension to a similar type series with an arbitrary number of numerator parameters and denominator parameters are basic hypergeometric series. We also have basic bilateral series. Most of the applications of the q concept have occurred in the field of pure mathematics, particularly in number theory, modular equations and elliptic integrals.

About two-thirds of the volume is devoted to the Gauss function and its generalizations as previously noted, and the remainder to basic hypergeometric series. A survey of the contents follows. Chapter 1 treats the Gauss function. A historical development of the function is traced. Some of the topics discussed are Kummer's 24 solutions, integral representations and analytic continuation.

The generalized Gauss function is taken up in Chapter 2. Here there is considerable material on the ${}_3F_2$ of unit argument and other special summation formulas. Products of hypergeometric series are also considered. A generalization of the ${}_pF_q$ known as Meijer's G-function is referenced but not discussed.

Barnes-type integral representations for the ${}_pF_q$ and other contour integrals for the ${}_pF_q$ are studied in Chapter 4. Integral transforms of the ${}_pF_q$ such as those of Euler and Mellin are also presented. Chapter 6 deals with bilateral series. Appell series and Lauricella functions are analyzed in Chapter 8.

Basic hypergeometric functions, basic hypergeometric integrals, basic bilateral series and basic Appell series are the subjects of Chapters 3, 5, 7 and 9, respectively. There are five appendices. The first gives properties of the gamma function expressed as a Pochammer symbol. Properties of the analog of this which appear in basic series are delineated in Appendix 2. Appendix 3 contains a list of formulas where the ordinary hypergeometric series for special arguments can be expressed as the product of gamma functions. These are called summation theorems. Summation theorems for basic series are listed in Appendix 4. Appendix 5 gives a table of $[\prod_{n=0}^{\infty} (1 - aq^n)^{-1}]$ for $q = 0(0.05)1.0$, $a = -0.9(0.05)0.9$ mostly to 9S. There is also a table of the above function for $a = 1$ and $q = 0(0.005)0.890$, mostly to 9S.

The bibliography is excellent, though nearly all the entries are dated before 1956. An index of symbols and a general index are provided.

Our casual reading has revealed several typographical errors. In formula (1.1.1.3) on p. 3,. $(a)_n$ should be replaced by $(a)_n/n^a\Gamma(n)$. On the same page in (1.1.8), ∞ in the integral limits should read $i\infty$. On p. 17, the right-hand sides of (1.5.6) and (1.5.8) should read $(\sin az)/a \sin z$ and $(\sinh az)/a \sinh z$, respectively. These and possibly other imperfections aside, the volume contains a wealth of information and should be most useful to pure and applied workers.

Y. L. L.